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"FUZZYMOS": A FUZZY LOGIC SYSTEM FOR OBJECTIVE AVIATION FORECASTING

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ABSTRACT

Verification statistics indicate Model Output Statistics (MOS) to be the best objective forecast tool for the aviation forecaster. However, a significant limitation is that the MOS categorical forecasts cover a broad range of ceiling and visibility values and can therefore be difficult to use operationally. This forces the aviation forecaster to use other means to determine the "best" ceiling height or visibility value within a particular category. FuzzyMOS is an objective forecast system that uses issue hour, forecast period, initial ceiling or visibility category, MOS, and verified MOS data as inputs to provide a categorical forecast output that is essentially a type of conditional and calibrated MOS forecast. This output is expressed as a decimal value and can then be converted to a specific ceiling height or visibility. Over 130,000 forecasts were used in this study, and test results indicate fuzzyMOS forecasts not only improved over MOS, but improved to a substantially greater degree than the official NWS forecasts.

FuzzyMOS is a fuzzy system, based on fuzzy logic. And since many readers may be unfamiliar with this topic, an introductory discussion of fuzzy logic is presented. This is followed by details of the fuzzyMOS system and its subsequent testing and verification.

1. Introduction

a. The trouble with MOS

Model Output Statistics (MOS, pronounced "moss") is the best objective forecast tool produced by the National Weather Service (NWS) for the aviation forecaster. Verification data from over 130,000 forecasts used in this study clearly indicate this fact. Forecasts based on conditional climatology from the PP-Tools software (Hicks, 1995) ran a close second in verification scores, but MOS was the overall winner.

But there is a problem with MOS. Aviation forecast output from MOS is in the form of one-digit categories--1 through 7 for ceiling, and 1 through 5 for visibility. These can be thought of as mathematical sets that contain forecast ceiling or visibility conditions that meet certain threshold values. For example, MOS category 4 for ceiling contains all ceiling heights from 1000 feet through 3000 feet. Similarly, MOS category 4 for visibility contains all visibility values from 3 miles through 5 miles. This is the problem. If MOS is forecasting a category 4 ceiling, should the forecaster be thinking about a 1000 foot ceiling, a 2000 foot ceiling, a 3000 foot ceiling, or what? In terms of operational impact, a forecast ceiling of 1000 feet is for conditions that are just barely VFR (visual flight rules), and this is sharply different operationally from a forecast ceiling of 3000 feet.

b. A different approach

This paper offers a different approach to objective aviation forecasting that builds on the skill of "calibrated" MOS data and ultimately produces a "fuzzy" categorical forecast that can easily be converted to a specific ceiling or visibility forecast. User inputs to this system are issue hour, forecast period, initial ceiling or visibility condition, and MOS forecast. An additional input is a disk file of verified MOS data. The "fuzzyMOS" output is a ceiling or visibility MOS forecast category--but expressed as a decimal number such as 4.3. This value is then converted to a ceiling or visibility forecast such as 2500 feet or 2.5 miles. Verification of fuzzyMOS forecast data for a six-month period for 30 forecast locations across the United States indicated that fuzzyMOS not only improved over MOS by 12.9 percent, but improved over MOS *to a greater degree* than the official NWS aviation forecast (8.0 percent over MOS)!

Fuzzy logic, or more specifically, a fuzzy logic system was the basis for this study. And since many readers may be unfamiliar with this topic, a brief discussion of fuzzy logic and its somewhat colorful history is presented. This will then be followed by a discussion of the development of fuzzyMOS and a look at its verification data.

2. Fuzzy logic and fuzzy systems

Fuzzy logic is not logic that is fuzzy, but logic that describes and tames fuzziness (McNeill and Freiberger, 1993). Since the days of the ancient Greeks, scientists and mathematicians have used "black and white" laws of logic to describe and discuss the "gray" universe. For centuries, many scientists, mathematicians and philosophers have brooded about this grayness (Kosko, 1993).

> *So far as the laws of mathematics refer to reality, they are not certain. And so far as they are certain, they do not refer to reality.*
>
> Albert Einstein
> Geometry and Experience

a. The fuzzy principle

The fuzzy principle states that *everything is a matter of degree* (Kosko, 1993). Fuzzy logic is based on the premise that any given object might belong to a certain "fuzzy" set to a degree ranging from 0 to 1. In practical terms, this might mean that a glass of water might be considered full to the degree 0.5. Or a person might be considered tall to the degree 0.8. Furthermore, an object might simultaneously belong to more than one such fuzzy set. For example, a glass of water might be considered empty to the degree 0.5 and at the same time be considered full to the degree 0.5. While this is no problem for fuzzy logic, it represents a paradox for conventional logic. And according to Kosko (1993), science has avoided this paradox by simply "rounding off" and calling the glass of water either full or empty.

The concept of fuzzy or "vague" sets was first introduced by Max Black (1909-1988), but it was Lotfi Zadeh, Professor of Systems Theory at the University of California, Berkley who actually developed and expanded the theory and made fuzzy logic a reality.

b. Controversy

Zadeh's paper "Fuzzy Sets" was published in June of 1965, but it actually came under fire even before it was published (McNeill and Freiberger,1993). According to Kosko (1993), "the term 'fuzzy' invited the wrath of science and received it. It forced the new field to grow up with all the problems of a 'boy named Sue.' Government agencies gave no grants for fuzzy research. Few journals or conferences accepted fuzzy papers. Academic departments did not promote faculty who did fuzzy research, at least who did only fuzzy research. The fuzzy movement in those days was a small cult and it went underground. It grew and matured without the usual support of subsidized science."

The concept of fuzzy sets with varying degrees of membership differs sharply from traditional mathematical theory because it violates laws of logic that date back to the Greek philosopher Aristotle. In particular, fuzzy logic breaks both the Law of Contradiction and the Law of the

Excluded Middle. The Law of Contradiction forbids both true and not-true at once. In the *Metaphysics*, Aristotle states it more carefully: "The same thing cannot at the same time both belong and not belong to the same object and in the same respect." The Law of the Excluded Middle, or more properly, the Law of Bivalence forbids anything other than true and not-true. In the *Metaphysics*, Aristotle expresses it thus: "Of any subject, one thing must be either asserted or denied." In other words, a sheep is either white or not-white. A statement is either true or not-true (McNeill and Freiberger, 1993).

McNeill and Freiberger also indicate that criticism of fuzzy logic was often harsh, as indicated by the following quotes:

> *Fuzzy theory is wrong, wrong, and pernicious. What we need is more logical thinking, not less. The danger of fuzzy logic is that it will encourage the sort of imprecise thinking that has brought us so much trouble. Fuzzy logic is the cocaine of science.*
>
> Professor William Kahan
> University of California at Berkeley

> *"Fuzzification" is a kind of scientific permissiveness. It tends to result in socially appealing slogans unaccompanied by the discipline of hard scientific work and patient observation.*
>
> Professor Rudolf Kalman
> University of Florida at Gainesville

> *Fuzziness is probability in disguise. I can design a controller with probability that could do the same thing that you could do with fuzzy logic.*
>
> Professor Myron Tribus
> IEEE Institute, May 1988

> *Fuzzy logic is based on fuzzy thinking. It fails to distinguish between the issues specifically addressed by the traditional methods of logic, definition and statistical decision-making.*
>
> Jon Konieki in *AI Expert* (1991)

Lotfi Zadeh credits these views to the "hammer principle." It says, if you have a hammer in your hand, and it's your only tool, everything starts to look like a nail (von Altrock, 1995). According to Kosko (1993), Zadeh's favorite quote was: "Friends come and go but enemies accumulate."

While Western science largely rejected fuzzy logic, the Japanese not only welcomed it, but dizzily embraced it by developing products such as intelligent washing machines, microwaves, cameras, camcorders, and automobiles (McNeill and Freiberger, 1993). What was needed in the West was a champion of the fuzzy cause.

c. Bart Kosko--an American samurai

McNeill and Freiberger use the term "American samurai" in reference to the fuzzy logic efforts of Professor Bart Kosko of the University of Southern California. Kosko's work in fuzzy theory can only be described as remarkable. With degrees in philosophy, economics, mathematics, and electrical engineering, he too brooded over the application of black-white science to a gray world. He earned his doctorate as a student of Lotfi Zadeh, and after witnessing some of the criticism of fuzzy logic, he decided to determine once and for all whether fuzzy logic had substance. If it existed, he would pursue it out to the end. If not, he would attack it with more vengeance than its worst critics. He threw himself utterly into fuzziness and not only proved it was real, but developed a new and all-embracing model of the field. Its base, which he calls the Subsethood Theorem, led him to many of the root concepts of math in general and probability in particular.

As described further by McNeill and Freiberger, "Kosko is a singular individual, brilliant, brash, self-disciplined, competitive, and highly controversial, even within the fuzzy community. He is a 33-year-old polymath at USC, a mathematician, engineer, black-belt karate expert, screenwriter, novelist, bodybuilder, and composer of symphonies and sonatas. In his spare time, he scuba dives, shoots trap, and hunts wild boar with bow-and-arrow."

> *Zadeh is a much nicer guy than I am. He won't argue with other people's rules. But that doesn't convince the mathematicians. You have to step into the ring, wear their boxing gloves, beat them at their own game. And that's where I come in.*
>
> *I have results I can prove and I throw an open challenge: I'll fight anyone, on any conditions, any terms, provided it's convenient. I have theorems and I have proofs. You've just got to take it on the chin. There's nothing you can do about it. I didn't walk into battle without a sword on.*
>
> Bart Kosko

Kosko's theorems have become the cornerstone of fuzzy theory. His *Fuzzy Entropy Theorem* (Kosko, 1992) was his breakthrough concept. Entropy is a measure of uncertainty or disorder in a system. His theorem measured the entropy or fuzziness of a fuzzy set, and it also went on to show that fuzziness actually begins where conventional or Western logic ends. Additionally, it yielded derivation of the fuzzy set rules of intersection, union, and complement that had previously been proposed by Zadeh.

The fuzzy entropy theorem paved the way for his *Subsethood Theorem* (Kosko, 1992). This very important theorem not only proved that fuzziness was real, but it derived Bayes theorem of probability as a special case of subsethood--or fuzziness. Before the subsethood theorem, Kosko says, probability rested on relative frequency as an axiom, beneath which lay only intuition. Now he says "the probabilist's axioms are the fuzzy theorist's theorems" (McNeill and Freiberger, 1993).

Kosko's *Fuzzy Approximation Theorem* (Kosko, 1993) serves as a foundation for fuzzy systems. It proves that a fuzzy system can model or approximate *any* system. The idea of a fuzzy system is that each piece of human knowledge can be expressed by rules of the form *IF this THEN that.* But the key to fuzzy systems is that the rules are *fuzzy* rules that relate to fuzzy sets. The better the rules cover the curve (or whatever) being modeled, then the smarter the system. And furthermore, all the rules fire--all the time. They fire in parallel, and all rules fire to some degree. Most rules fire to zero degree, some only fire partially. The result is a fuzzy weighted average that determines the system's output.

Three very important characteristics of fuzzy systems are worthy of further emphasis: (1) fuzzy systems are essentially math-free, (2) it does not matter if the variables are independent and normally distributed, as often required by traditional statistical theory, and (3) the relationship between inputs and output can take any form--it need not be linear.

3. Development of the fuzzyMOS system–adapted from von Altrock (1995)

An overview of the fuzzyMOS system is shown in Figure 1. There are four user inputs: issue hour, forecast period, initial ceiling or visibility MOS category (Cat0), and the MOS categorical forecast. An additional input to the system is a previously processed file of verified MOS data. Three important steps in the development of a fuzzy logic system are fuzzification, fuzzy inference, and defuzzification. Each of these topics will now be examined in more detail.

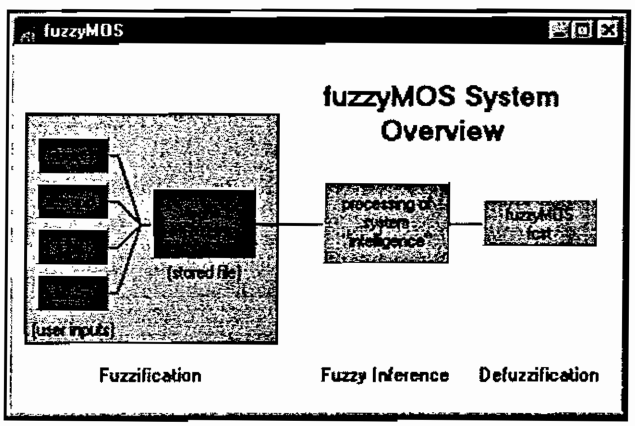


Figure 1. Overview of the fuzzyMOS system.

a.Fuzzification

The first step in developing a fuzzy system is to define "linguistic variables" that use descriptive and subjective terms to describe the inputs and output to the system. This is the "fuzzification" step. Linguistic variables typically use words rather than rigid numbers to describe an object. As an example, the linguistic variable "distance" could have the linguistic terms {far, medium, close, zero, too_far}. Each of these descriptive terms is actually a fuzzy set that for any given situation has a membership value between 0 and 1. For example, a given distance could be expressed as far to the 0.6 degree and medium to the 0.4 degree. The linguistic variables and terms for the fuzzyMOS system are as follows:

linguistic variable	**possible values (terms)**
IssueHr	{00z, 06z, 12z, 18z}
Period	{3hr, 6hr, 9hr, 12hr}
Cat0	{1, 2, 3, 4, 5, 6, 7}
MOS	{1, 2, 3, 4, 5, 6, 7}
fuzzyMOS	{1, 2, 3, 4, 5, 6, 7}

The fuzzification step is not complete until the membership function for each term has been defined. The membership degree for any term ranges from 0 to 1, with the maximum degree associated with the term's most typical value. Adjacent terms should overlap with one term's maximum membership at the same point where the next term has a membership of zero. An example of the completed membership functions for the five terms in the linguistic variable IssueHr is shown in Figure 2.

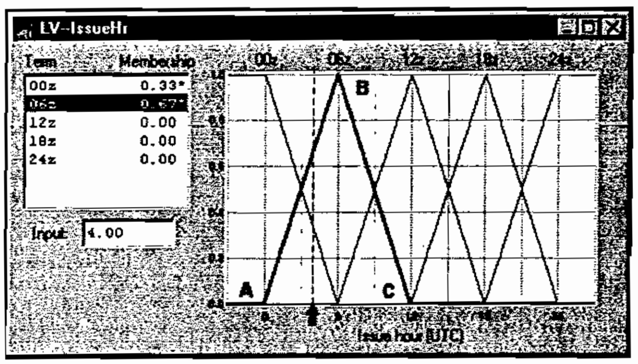


Figure 2. Linguistic variable IssueHr and its terms.

The membership function for the 06z term is highlighted in Figure 2. This function can be defined and subsequently evaluated by specifying three points in x-y terms. Point A(0,0) indicates zero membership at IssueHr 0, point B (6,1) represents maximum membership at IssueHr 6, and point C(12,0) represents zero membership at IssueHr 12. Note that for an input hour of 4, the degree of membership in the 06z term is 0.67. And very importantly, note that the input hour of 4 also has membership in the 00z term to the degree of 0.33. For any value of IssueHr, it will always have membership in either one or two fuzzy sets represented by the specified linguistic terms.

In fuzzy logic, there are four so-called "standard membership functions." These are "Z-type", "lambda-type", "pi-type" (for the Greek letter π), and "S-type." Each term's name is based on its graphic appearance. In Figure 2, the 00z term is the Z-type, the 24z term is the S-type, and the remaining terms are of the lambda type. The advantage of these standard membership functions is that they can quickly be evaluated by computer processing. The computer code need only store two to four points and one to two slope values for each term. The value of the membership function can then be determined by an algorithm such as:

```
Slope1 = (B.y-A.y)/(B.x-A.x)     // previously computed and stored
Slope2 = (C.y-B.y)/(C.x-B.x)     // previously computed and stored

IF input <= A.x THEN membership = 0
ELSE IF input <= B.x THEN membership = min{1,(input-A.x)*Slope1}
ELSE membership = max{0, 1-(input-PointB.X)*-Slope2)}.
```

The remaining linguistic input variables for the fuzzyMOS system are shown in Figures 3-5. The output linguistic variable will be discussed later, in the "defuzzification" part of the fuzzyMOS process. It should be pointed out that each of the graphics in this paper are output from a computer program developed by the author for the purpose of testing and demonstrating fuzzyMOS. This computer program was developed in Borland Delphi 2, and its functionality is based on ideas from the *fuzzyTech* software that is packaged with the reference text *Fuzzy Logic and NeuroFuzzy Applications Explained* by von Altrock.

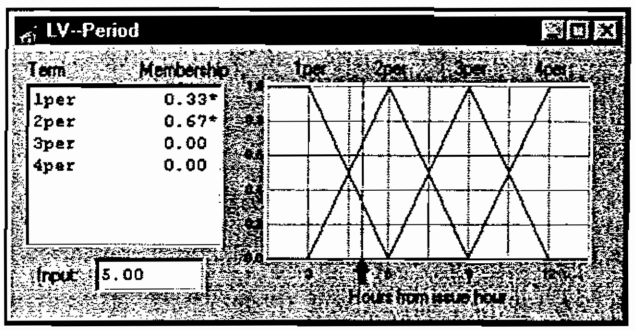


Figure 3. Linguistic variable Period and its terms.

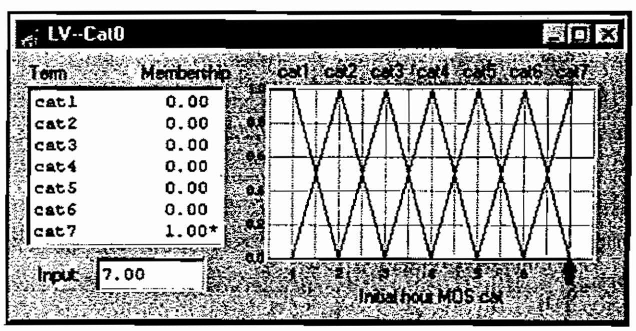


Figure 4. Linguistic variable Cat0 and its terms.

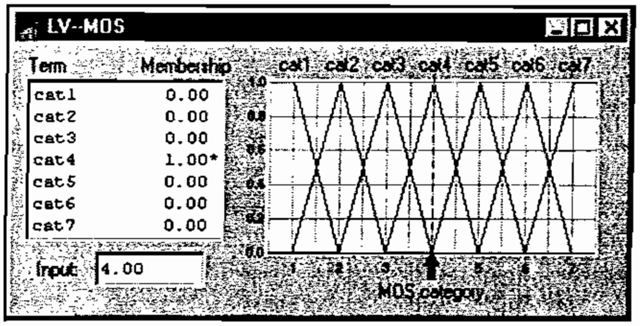


Figure 5. Linguistic variable MOS and its terms.

The linguistic variables that have been defined are actually used as links to the verified MOS data that serves as the final and most important input to the fuzzyMOS system. This verification data gives the fuzzyMOS system the "intelligence" that is processed into the final forecast output.

A computer program developed by the author was used to gather forecast verification data for thirty forecast locations in the United States. The stations used in the study are listed in Table 1.

ABI	Abilene, Texas	LBB	Lubbock, Texas
ABQ	Albuquerque, New Mexico	LIT	Little Rock, Arkansas
ACT	Waco, Texas	MAF	Midland-Odessa, Texas
AMA	Amarillo, Texas	MCI	Kansas City, Missouri
ATL	Atlanta, Georgia	ORL	Orlando, Florida
BOS	Boston, Massachusetts	MEM	Memphis, Tennessee
BRO	Brownsville, Texas	MIA	Miami, Florida
CLT	Charlotte, North Carolina	MSY	New Orleans, Louisiana
CRP	Corpus Christi, Texas	OKC	Oklahoma City, Oklahoma
DCA	Washington, DC.	PHX	Phoenix, Arizona
DEN	Denver, Colorado	SAT	San Antonio, Texas
DFW	Dallas-Fort Worth, Texas	SHV	Shreveport, Louisiana
FTW	Fort Worth, Texas	SJT	San Angelo, Texas
IAH	Houston, Texas	SLC	Salt Lake City, Utah
JAX	Jacksonville, Florida	SPS	Wichita Falls, Texas

Table 1. Forecast verification locations used for fuzzyMOS rules

For each forecast location, aviation terminal forecasts, which may be referred to as terminal or aerodrome forecasts (TAFs) are issued four times daily: 00 UTC, 06 UTC, 12 UTC, and 18 UTC. The verification data for each station consisted of the MOS categorical forecast, a categorical conditional climatology forecast from PP-Tools, and the observed MOS categorical forecast. The verification data were gathered for four forecast periods: 3, 6, 9, and 12 hours from the valid hour of the forecast. Six months of forecast verification data were used to develop the fuzzyMOS rules: January-June 1997.

The verification data were used to create tables for ceiling and visibility forecasts--one for each combination of issue hour {0, 6, 12, 18 UTC}, period {3, 6, 9, 12 hours}, and initial MOS category {1, 2, 3, 4, 5, 6, 7}. For the six months of data, this resulted in 112 tables of ceiling forecast data and 80 tables of visibility forecast data such as depicted in Figure 6.

		obsvd cat						
		7	6	5	4	3	2	1
	7	2534	86	71	25	6	0	1
	6	162	61	26	9	3	1	0
	5	97	22	33	9	5	1	0
fcst cat	4	56	7	6	19	6	2	1
	3	2	2	2	5	2	3	0
	2	1	0	0	0	0	1	0
	1	1	0	0	1	0	0	0

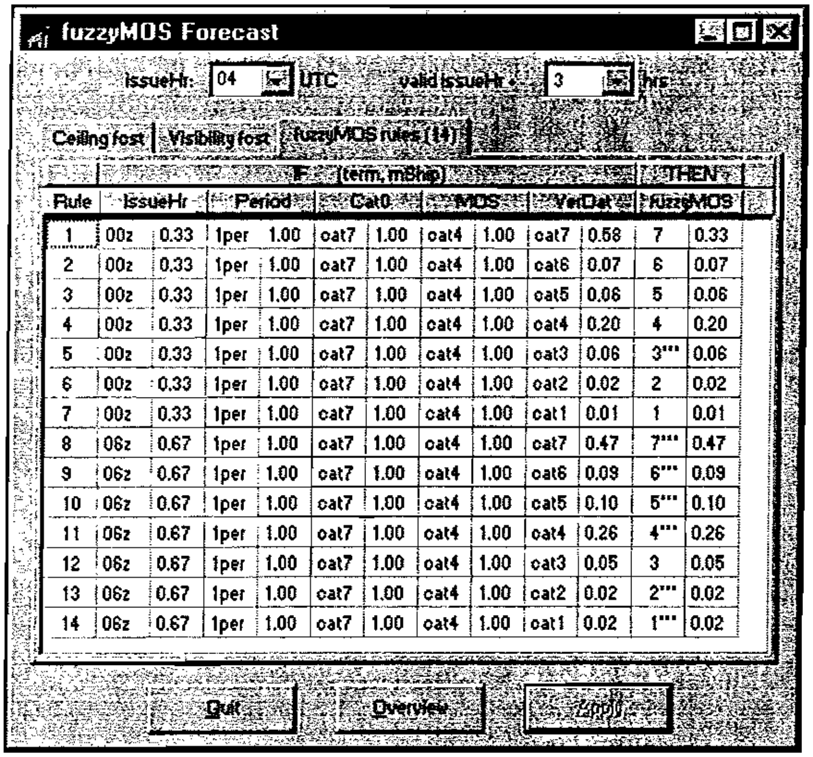
Figure 6. MOS verification (frequency) data for 00 UTC ceiling forecasts, valid 3 hours later, with initial ceiling category 7.

The illustrated table is for MOS ceiling forecasts issued at 00 UTC, valid 3 hours later, with an initial ceiling category of 7. The tabular frequency data shows that when MOS forecast a category 4 ceiling, the observed ceiling condition was most frequently a higher category 7. Of 97 forecasts verified, 56 resulted in category 7, while only 19 resulted in category 4. MOS forecasts for TAFs issued at 00 UTC would always be based on the previous 12 UTC model run and would therefore be 12 hours old. But when matched with an initial ceiling category of 7, the output indicates MOS was most likely to be pessimistic for this situation. So the data in the tables can be considered to be a type of conditional and "calibrated" MOS data. The tables should capture any systematic biases or weaknesses that MOS might have for a given forecast situation.

The fuzzification of the verification data is accomplished by simply dividing each observed frequency value for a given MOS forecast by the sum of the frequencies for all observed categories for that particular forecast. For example, using Figure 6 and a MOS forecast of category 4, the sum of all the observed frequencies would be 56 + 7 + 6 + 19 + 6 + 2 + 1 = 97. Therefore, the fuzzification of observed categories 7 through 1 when a category 4 was forecast by MOS would yield {0.58, 0.07, 0.06, 0.20, 0.06, 0.02, 0.01}. This represents the degree to which each possible category was actually observed for the given forecast situation.

b. Fuzzy inference using IF-THEN rules

Now that all input variables have been converted to linguistic variables, the "fuzzy inference" step is used to identify the rules that apply to the current situation and then compute the values of the output linguistic variable. Figure 7 shows an example of rules used in the fuzzyMOS system.



	IF (term, mShip)										THEN	
Rule	IssueHr		Period		Cat0		MOS		VerDat		fuzzyMOS	
1	00z	0.33	1per	1.00	cat7	1.00	cat4	1.00	cat7	0.58	7	0.33
2	00z	0.33	1per	1.00	cat7	1.00	cat4	1.00	cat6	0.07	6	0.07
3	00z	0.33	1per	1.00	cat7	1.00	cat4	1.00	cat5	0.06	5	0.06
4	00z	0.33	1per	1.00	cat7	1.00	cat4	1.00	cat4	0.20	4	0.20
5	00z	0.33	1per	1.00	cat7	1.00	cat4	1.00	cat3	0.06	3'''	0.06
6	00z	0.33	1per	1.00	cat7	1.00	cat4	1.00	cat2	0.02	2	0.02
7	00z	0.33	1per	1.00	cat7	1.00	cat4	1.00	cat1	0.01	1	0.01
8	06z	0.67	1per	1.00	cat7	1.00	cat4	1.00	cat7	0.47	7'''	0.47
9	06z	0.67	1per	1.00	cat7	1.00	cat4	1.00	cat6	0.09	6'''	0.09
10	06z	0.67	1per	1.00	cat7	1.00	cat4	1.00	cat5	0.10	5'''	0.10
11	06z	0.67	1per	1.00	cat7	1.00	cat4	1.00	cat4	0.26	4'''	0.26
12	06z	0.67	1per	1.00	cat7	1.00	cat4	1.00	cat3	0.05	3	0.05
13	06z	0.67	1per	1.00	cat7	1.00	cat4	1.00	cat2	0.02	2'''	0.02
14	06z	0.67	1per	1.00	cat7	1.00	cat4	1.00	cat1	0.02	1'''	0.02

Figure 7. Example of fuzzyMOS rules.

Rules for a fuzzy system can come from various sources, and they can be stated in simple terms. For example, rules for a control-type system could come from interviewing an expert: "IF distance is small THEN power = medium." Rules can come from other sources such as neural nets, where the system essentially learns its own rules. As a demonstration, Kosko(1992) and others developed a neurofuzzy "truck backer-upper system" that modeled backing up a truck and trailer rig (18-wheeler) from a random location in a parking lot to a loading dock. The system learned how to avoid "jackknife" situations and was able to smoothly back up to the dock. This particular system also clearly demonstrated how fuzzy systems could be used for modeling systems that would be extremely difficult (if not impossible) to model using conventional methods.

Rules for fuzzyMOS come from the MOS verification data. Each combination of issue hour, period, initial category, MOS category, and observed MOS category makes one rule. These rules represent the "intelligence" of a fuzzy system and are critical to the quality of the system's output. Fourteen rules apply to the example forecast. Rule 1 can be read as follows:

```
IF (IssueHr = 00z) AND (Period = 1per) AND (Cat0 = cat7)
AND (MOS = cat4) AND (VerDat = cat7) THEN fuzzyMOS = 7.
```

Each of the rules has five input conditions and one output. Each of the terms in the rule has a membership value that denotes the degree to which that term applies to this particular forecast situation. Using rule 1 as an example, the degree to which a category 7 was actually observed in the verification data was 0.58. Each rule also has an output weight associated with its fuzzyMOS term. This very important value represents the overall weight or degree of truth of the combined terms in the IF part of the rule. The determination of this output weight will be discussed next.

1) Aggregation: computation of the IF part of the rules

Referring back to rule 1 from Figure 7, the IF part of the rule combines the five conditions "IssueHr = 00z", "Period = 1per", "Cat0 = cat7, and "MOS = cat4" and "VerDat = cat7." This combination of terms actually defines whether the rule is valid in the current situation or not. In conventional logic, the combination of these terms would be computed by the Boolean AND operator. But in the case of fuzzy logic, the Boolean AND cannot be used because it cannot cope with conditions that are not either all-true or all-false. In the development of fuzzy logic theory, new set operators were derived for logical connectives such as AND, OR, and NOT. These fuzzy set operators are listed in Table 2.

μ_A AND μ_B	$= \min\{\mu_A, \mu_B\}$
μ_A OR μ_B	$= \max\{\mu_A, \mu_B\}$
NOT μ_A	$= 1 - \mu_A$

Table 2. Logic operators for fuzzy sets.

The aggregation step is completed by applying the fuzzy set AND operator to the membership values of the five conditions of the IF part of the rules from Figure 7 and the degree to which the specific MOS category was actually observed. The membership values used in this step are taken directly from Figures 2-5 and are also shown in each rule in Figure 7. The degree to which the forecast MOS category was actually observed is the fuzzified verification data from tables such as that shown in Figure 6. Results of the aggregation step for rules 1 through 7 are shown in Table 3.

Rule 1	$\min\{0.33, 1.00, 1.00, 1.00, 0.58\} = 0.33$
Rule 2	$\min\{0.33, 1.00, 1.00, 1.00, 0.07\} = 0.07$
Rule 3	$\min\{0.33, 1.00, 1.00, 1.00, 0.06\} = 0.06$
Rule 4	$\min\{0.33, 1.00, 1.00, 1.00, 0.20\} = 0.20$
Rule 5	$\min\{0.33, 1.00, 1.00, 1.00, 0.06\} = 0.06$
Rule 6	$\min\{0.33, 1.00, 1.00, 1.00, 0.02\} = 0.02$
Rule 7	$\min\{0.33, 1.00, 1.00, 1.00, 0.01\} = 0.01$

Table 3. Results of rule aggregation step.

The results of the aggregation now agree with the rule output weights indicated in Figure 7. But there is still more work to be done. As shown in Figure 7, a total of 14 rules are actually used.

For rules 8 through 14, the issue hour term is different as are the membership values for the MOS verification data. And very importantly, notice that some of the outputs are for the same fuzzyMOS category--but to different degrees. This leads to the composition step.

2) Composition: computation of the THEN part of the rules

Each rule defines an action to be taken in the THEN part of the rule. The degree to which the action is valid is given by the adequateness of the rule to the current situation. This adequateness has been computed by the aggregation step as the degree of truth of the IF part of the rule. For the example forecast, we have 14 rules that apply to seven possible outputs--to varying degrees. These outputs must be combined before proceeding to the defuzzification step..

In a fuzzy logic rule system, either rule 1 is true, OR rule 2 is true, OR rule 3 is true, OR.... Using the fuzzy logic operators as listed in Table 2, the OR is mathematically represented by the max operator. Referring back to the example rules shown in Figure 7, rules 1 and 8 both have outputs of fuzzyMOS category 7--but to different degrees. The final output weight for fuzzyMOS category 7 will be the maximum of these two values, 0.47. Applying this composition step to the seven possible outputs for the example forecast completes the fuzzy inference step of the fuzzyMOS process. The final outputs and their weights are shown in Table 4.

category 7	to the degree of 0.47	(= max{0.33, 0.47})
category 6	to the degree of 0.09	(= max{0.07, 0.09})
category 5	to the degree of 0.10	(= max{0.06, 0.10})
category 4	to the degree of 0.26	(= max{0.20, 0.26})
category 3	to the degree of 0.06	(= max{0.06, 0.05})
category 2	to the degree of 0.02	(= max{0.02, 0.02})
category 1	to the degree of 0.02	(= max{0.01, 0.02})

Table 4. Fuzzy inference results for linguistic variable fuzzyMOS in the example forecast.

Before leaving the fuzzy inference topic, it should be pointed out that there is a limit on the number of rules that can apply to any given forecast. Note that the verification data contains up to 4 x 4 x 7 x 7 x 7 = 5,488 possible rules for ceiling forecasts and up to 4 x 4 x 5 x 5 x 5 = 2,000 possible rules for visibility forecasts (based on the possible number of issue hours, periods, initial categories, MOS forecast categories and observed MOS categories). However, the membership functions for each variable's linguistic terms are structured in a way that allows no more than two terms to apply to issue hour, period, and initial MOS category. The nonfuzzy MOS forecast is always limited to just one term. This means that no more than 2 x 2 x 2 x 1 x 7 = 56 rules can apply to a particular ceiling forecast, and no more than 2 x 2 x 2 x 1 x 5 = 40 rules can apply to a visibility forecast.

c. Defuzzification

At the end of the fuzzy inference step, the result for fuzzyMOS is given as the value of a linguistic variable. The value of the fuzzyMOS linguistic variable is defined by the membership values of its seven linguistic terms. Before this information can be used in the preparation of a forecast, it must first be translated into a single numeric value. This step is called defuzzification.

The result of the fuzzy inference for the example shown in Table 4 is both fuzzy and ambiguous with seven different outputs having non-zero degrees of truth. How can these conflicting actions that are defined as fuzzy sets be combined to produce a single numeric value that can be used in the preparation of a forecast? The answer is that the various outputs are balanced. Most methods for defuzzification use a two-step approach. In the first step, a "typical" value is computed for each term in the linguistic variable. In the second step, the "best compromise" is determined by "balancing" the results. An example of this defuzzification process is shown in Figure 8.

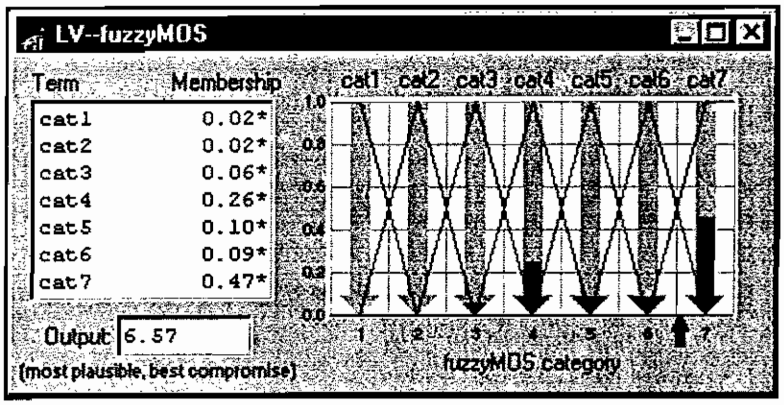


Figure 8. Example of fuzzyMOS defuzzification process.

The most common approach to computing the typical value of each term is simply to use the value associated with the maximum of the respective membership function. For the example shown here, the large arrows coincide with the typical values for each term. The part of each arrow that is black represents the membership value of that particular term. The computed balance point for the example was at a fuzzyMOS value of 6.57. But due to the nature of the verification data, one additional step was used to arrive at this value. This involves the concept of "most plausible value."

The distribution of observed categorical outputs is often somewhat discontinuous and bimodal, with higher frequencies associated with unlimited ceiling or visibility conditions and also at some lower category. This characteristic also appears in the fuzzyMOS output. The approach taken with fuzzyMOS was to first group the outputs around membership values that were above average (0.14 in the example). For the example shown here, that led to groups consisting of fuzzyMOS categories {7,6,5} and {6,5,4,3,2,1}. The membership values of each group were added together, and the group with the higher value was considered to be the "most plausible" group--{7,6,5} in this case. The "best compromise" technique was then used on this group.

The best compromise employs the "weight-arm" technique from physics and is computed by multiplying each term's typical value by its membership value and then dividing the sum of these values by the sum of the membership values. For the example, that leads to a numerator of (7 x 0.47) + (6 x 0.09) + (5 x 0.10) = 4.33 and a denominator of (0.47 + 0.09 + 0.10) = 0.66. This leads to a fuzzyMOS value of 4.33/0.66 = 6.56. The value of 6.57 shown in Figure 8 is of course more precise, since it was calculated with more than two-digit precision.

The final step in defuzzification is to convert the fuzzyMOS categorical forecast into a forecast of ceiling height or visibility. Since the fuzzyMOS output is a decimal value based on weighted output, it can be assumed to include more detail about the probable ceiling or visibility condition than contained in the single-digit MOS categorical forecast. The conversion of fuzzyMOS to real ceiling or visibility values is a straightforward linear transformation based on the boundaries of the MOS categories shown in Table 5. It should be noted that the boundary threshold values were adjusted slightly in order to better match the category boundaries of the fuzzyMOS linguistic terms.

Ceiling category	height (feet)	Visibility category	miles
7	> 12000	5	> 5
6	6600 - 12000	4	3 - 5
5	3100 - 6500	3	1 - 2 3/4
4	1000 - 3000	2	1/2 - 7/8
3	500 - 900	1	< 1/2
2	200 - 400		
1	< 200		

Table 5. MOS categories for ceiling and visibility forecasts.

The algorithm for converting ceiling forecasts is as follows:

```
IF fuzzyMOS > 6.5 THEN
  value = 9999              // unlimited ceiling
ELSE IF fuzzyMOS > 5.5 THEN
  value = 6550 + (fuzzyMOS-5.5)*(11500-6550)
ELSE IF fuzzyMOS > 4.5 THEN
  value := 3050 + (fuzzyMOS-4.5)*(6550-3050)
ELSE IF fuzzyMOS > 3.5 THEN
  value = 950 + (fuzzyMOS-3.5)*(3050-950)
ELSE IF fuzzyMOS > 2.5 THEN
  value := 450 + (fuzzyMOS-2.5)*(950-450)
ELSE IF fuzzyMOS > 1.5 THEN
  value = 150 + (fuzzyMOS-1.5)*(450-150)
ELSE value = 100.
```

Similarly, the algorithm for converting visibility forecasts is:

```
IF fuzzyMOS > 4.5 THEN
  value = 7                    // unlimited visibility
ELSE IF fuzzyMOS > 3.5 THEN
  value = 2.875 + (fuzzyMOS-3.5)*(5.5-2.875)
ELSE IF fuzzyMOS > 2.5 THEN
  value = 0.875 + (fuzzyMOS-2.5)*(2.875-0.875)
ELSE IF fuzzyMOS > 1.5 THEN
  value = 0.375 + (fuzzyMOS-1.5)*(0.875-0.375)
ELSE value = 0.25.
```

For the example forecast, the ceiling transformation algorithm converts the fuzzyMOS forecast of 6.57 to a forecast ceiling height of 9999 which denotes an unlimited ceiling. But one final addition was made to the defuzzification process that provides more information to the aviation forecaster in certain instances. If the combined membership of a secondary lower group in a bimodal output distribution equals 0.50 or higher (as in this case), this indicates a fairly high possibility that the lower condition might occur. The fuzzyMOS textual forecast therefore includes that group's output value preceded by the word "TEMPO." For the example forecast the textual forecast for ceiling height was "unl TEMPO cig024." If the combined membership of the lower group is less than 0.50, then the output ceiling height for that lower cloud group would be preceded by "SCT", with a textual forecast for ceiling height such as "100 SCT024." This provides more detail to the forecaster about the uncertainty of the particular forecast situation. This concludes the defuzzification step, and this also concludes the development of the fuzzyMOS process.

4. Testing and verification of fuzzyMOS

a. Tests on developmental data

As mentioned previously, the fuzzyMOS system was developed from verification data for 30 forecast locations, using forecasts issued at 00 UTC, 06 UTC, 12 UTC, and 18 UTC and subsequently verified 3, 6, 9, and 12 hours after issuance. The data used was for the period January through June of 1997. This resulted in 137,416 MOS forecasts that were used to create 192 verification tables such as described earlier in Figure 6. The first test of fuzzyMOS was made on this developmental data, and fuzzyMOS was the winner in every forecast period. The results are summarized in Table 6. It should be pointed out that for any single forecast, "category error" is defined as the absolute value of the forecast MOS category minus the observed MOS category. The values in the table represent the total of those errors.

	Ceiling Category Errors				Visibility Category Errors			
	3hr	6hr	9hr	12hr	3hr	6hr	9hr	12hr
Persistence	10136	12652	14505	14285	2777	3652	3793	3819
PP-Tools	10117	11851	13323	13073	2351	2861	2644	2624
MOS	11615	11316	11581	10573	3402	3586	3469	3253
TAF	9633	10562	11361	10854	2529	2937	3003	2968
fuzzyMOS	**9246**	**9902**	**10458**	**9570**	**2148**	**2524**	**2397**	**2426**

Table 6. Verification results for fuzzyMOS developmental data, January 1997-July 1997. Lowest category errors are shown in bold type.

When developing a model from a set of data and then subsequently testing the model on that same data, there is always the risk that the model might work just fine on the developmental data but not work very well on independent data. To avoid this situation, the verification data was separated into two groups: forecasts issued on even days and forecasts issued on odd days. The fuzzyMOS tables were then developed from forecasts issued on odd days and then tested on forecasts issued on even days. Then this strategy was repeated with the tables being developed from forecasts issued on even days and subsequently tested on forecasts issued on odd days. The combined results of these tests are summarized in Table 7. FuzzyMOS was clearly the winner, with lowest category errors in all four periods of ceiling forecasts and three of four periods for visibility forecasts. FuzzyMOS improved over MOS a total of 12.9%, while the official TAF improved over MOS 8.0%. The comparative results are perhaps more apparent in the graph of Figure 9. Comparative improvements over persistence and over MOS are indicated in Table 8.

	Ceiling Category Errors				Visibility Category Errors				Total
	3hr	6hr	9hr	12hr	3hr	6hr	9hr	12hr	
Persistence	10067	12592	14410	14224	2760	3609	3737	3763	65162
PP-Tools	10051	11792	13230	13010	2330	2831	2604	**2584**	58432
MOS	11462	11239	11508	10521	3355	3550	3412	3208	58255
TAF	9582	10533	11321	10816	2508	2912	2969	2929	53570
fuzzyMOS	**9453**	**10221**	**10920**	**10079**	**2283**	**2692**	**2520**	2594	**50762**

Table 7. Combined verification results for the odd-day and even-day test data of January-June 1997. Lowest category errors are shown in bold type. Overall improvement over MOS was 12.9% for fuzzyMOS and 8.0% for TAF.

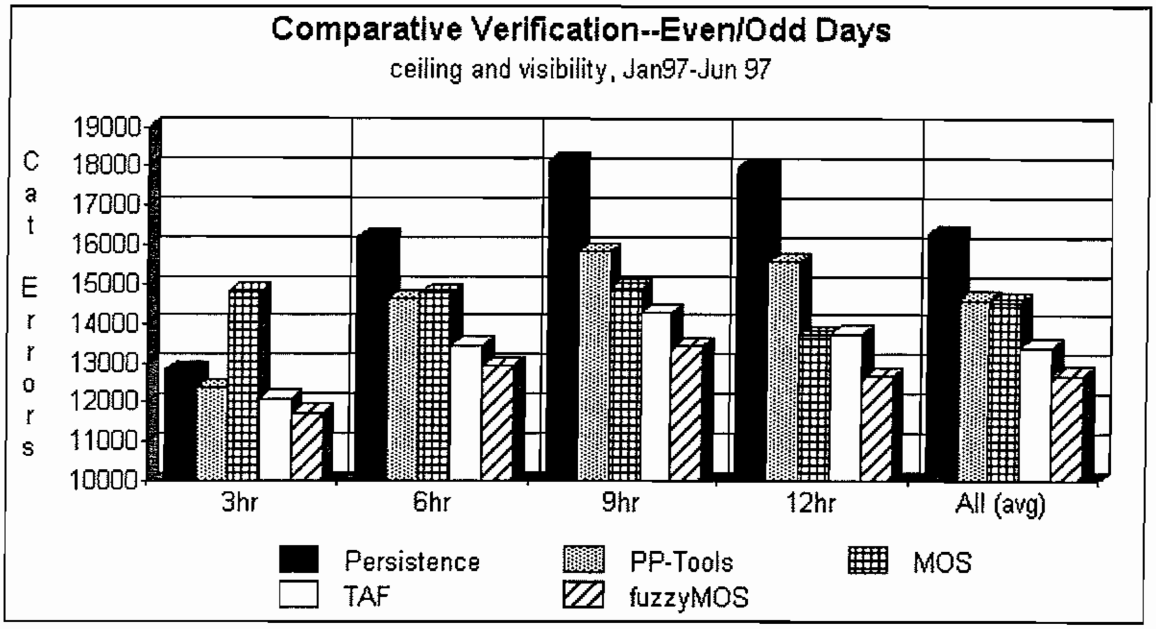


Figure 9. Comparative forecast verification of combined even-day and odd-day forecasts.

Comparative Forecast Improvement

	over Persistence	over MOS
PP-Tools	10.3%	
MOS	10.6%	
TAF	17.8%	8.0%
fuzzyMOS	22.1%	12.9%

Table 8. Comparative forecast improvement, even-day and odd-day forecasts.

Using the same odd-day and even-day test data, a more detailed analysis was made to determine how many times the original MOS forecast was changed and the success rate of these changes. These results are summarized in Table 9. In every forecast period, when fuzzyMOS differed with MOS, it most often resulted in an improved forecast. This table also indicates that the degree of improvement over MOS was much greater with visibility forecasts than with ceiling forecasts. Furthermore, this much greater improvement resulted from changes to a much smaller percentage of MOS forecasts.

	Ceiling forecasts				Visibility forecasts			
	3hr	6hr	9hr	12hr	3hr	6hr	9hr	12hr
fcsts	17989	17382	17374	15802	17946	17378	17371	15791
better	18%	15%	13%	13%	7%	8%	7%	7%
worse	11%	10%	10%	10%	3%	4%	3%	4%
same error	1%	1%	1%	1%	1%	0%	1%	1%
unchanged	71%	75%	77%	77%	90%	89%	90%	89%
error ipvmt	18%	9%	5%	4%	49%	39%	42%	32%

Table 9. Comparison of fuzzyMOS versus MOS for the combined odd-day and even-day test data of January-June 1997.

b. Additional test on independent data

An additional test of fuzzyMOS was made on an independent set of verification data for NWS aviation forecasts issued for the one-year period July 1996 through June 1997. The forecast locations for this data set are shown in Table 10, and it should be noted that PP-Tools forecasts were not available for this data. The test results are shown in Table 11, Figure 10, and Table 12. FuzzyMOS was the winner in every forecast period--and to an even greater degree overall.

ABI	Abilene, Texas
ACT	Waco, Texas
DAL	Love Field, Dallas, Texas
DFW	Dallas-Fort Worth, Texas
LFK	Lufkin, Texas

Table 10. Forecast locations for independent test data, July 1996 - June 1997.

	Ceiling Category Errors				Visibility Category Errors				Total
	3hr	6hr	9hr	12hr	3hr	6hr	9hr	12hr	
Persistence	2089	2608	3141	3178	446	631	648	688	13429
PP-Tools	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
MOS	2333	2255	2321	2231	595	633	610	605	11583
TAF	2164	2313	2432	2347	461	576	519	584	11396
fuzzyMOS	**1892**	**1988**	**2166**	**1995**	**340**	**415**	**388**	**411**	**9595**

Table 11. Verification results for the independent data set of July 1996-June 1997. Lowest category errors are shown in bold type.

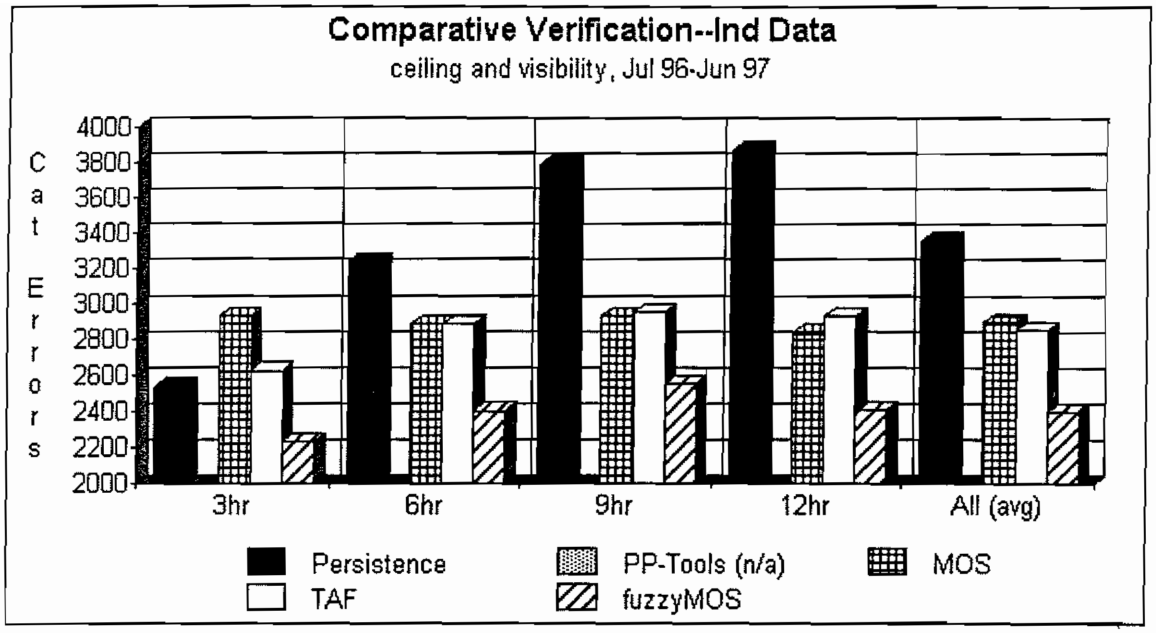


Figure 10. Comparative forecast verification of independent test data of July 1996-June 1997.

Comparative Forecast Improvement	over Persistence	over MOS
PP-Tools	n/a	
MOS	13.7%	
TAF	15.1%	1.6%
fuzzyMOS	28.6%	17.2%

Table 12. Comparative forecast improvement, independent test data of July 1996-June 1997.

5. Conclusions

All test results indicate that fuzzyMOS forecasts demonstrated a consistent improvement over MOS forecasts. Furthermore, the demonstrated improvement over MOS was to a substantially greater degree than the official NWS aviation forecasts. The consistent improvement over MOS in combination with the more-detailed ceiling and visibility forecasts indicate that fuzzyMOS could be a valuable tool for the preparation of aviation forecasts. And the implications of a forecast system that might potentially outperform aviation meteorologists are staggering.

This fuzzyMOS forecast system will continue to undergo testing and tuning, and the database from which it was developed will continue to grow. Additional ideas that will be tested are (1) fuzzyMOS databases based on more regionalized forecasts and (2) fuzzyMOS databases that are seasonal. Perhaps a combination of these two ideas can lead to even better forecast results.

This fuzzyMOS forecast system will be used by another computer program currently under development by the author. This program is called FREDD (Forecasting Resource and Evaluative Data Display) and will include computer-generated aviation terminal forecasts that will also be automatically monitored, automatically amended, and comprehensively verified. FREDD will employ an additional level of artificial intelligence that will base its aviation forecasts not only on fuzzyMOS but also on surface observations, conditional climatology, and WSR-88D Doppler radar information. FREDD should be completed and ready for testing by Fall of 1997.

Results of this study suggest that fuzzy systems might have great potential for other types of meteorological modeling. As mentioned earlier, fuzzy systems do not require independent and normally-distributed variables, and the relationships between inputs and output need not be linear. Additionally, vast data resources are often available for meteorological modeling, and this would allow for the development of fuzzy rules required by any such system.

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